Algorithms for Nearest Neighbors

Background and Two Challenges

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Outline

1. Formulating the Problem
2. Nearest Neighbors for Texts
3. Proving Hardness of Nearest Neighbors

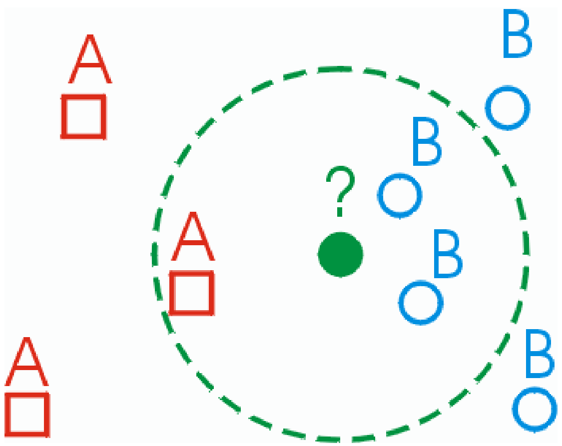
Part I
Formulating the Problem

Informal Problem Statement

To preprocess a database of *n* objects
so that given a query object,
one can effectively determine
its nearest neighbors in database

First Application (1960s)

Nearest neighbors for classification:



Picture from http://cgm.cs.mcgill.ca/ soss/cs644/projects/perrier/Image25.gif

Applications

What applications of nearest neighbors do you know?

- Text classification
- Statistical data analysis, e.g. medicine diagnosis
- Pattern recognition: characters, faces
- Code plagiarism detection
- Coding theory
- Data compression
- **Web:** recommendation systems, on-line ads, personalized news aggregation, long queries in web search, near-duplicates detection

Data Model in General

Formalization for nearest neighbors consists of:

- Representation format for objects
- Similarity function

Basic Data Models (1/2)

- Vector Model
  - Similarity: l^2, scalar product, cosine
- String Model
  - Similarity: Hamming distance, edit distance
- Black-box model
  - Similarity: given by oracle
    The only knowledge is triangle inequality

Basic Data Models (2/2)

- Set Model
  - Similarity: size of intersection
- Small graphs
  - Similarity: structure/labels matching

Algorithmic Approaches to NN

- Divide and conquer
- Traversal techniques
- Look-up techniques
- Contractive and low-distortion embeddings
- Tournament algorithms

Part II
Nearest Neighbors for Texts

Sparse Vector Model

Database: points in R^d,
every point has at most $k \ll d$ nonzero coordinates

Similarity: scalar product

Constraints:
$poly(n+d)$ for preprocessing time,
$poly(k) \cdot polylog(n+d)$ for query

Open Problem: solve NN for sparse vector model within given constraints

Inverted Index

Preprocessing:
For very term store a list of all documents in database with nonzero weight on it

Query processing:
Retrieve all point that have at least one common term with the query documet;
Perform linear scan on them

Rare-Point Method

Cheating: we will search only for neighbors that have at least one common rare feature with query object

Preprocessing:
For very rare feature store a list of all objects in database having it

Query processing:
Retrieve all point that have at least one common rare feature with the query object;
Perform linear scan on them

Probabilistic Analysis in a Nutshell

- We define a probability distribution over databases
- We define probability distribution over query objects
- We construct a solution that is efficient/accurate with high probability over "random" input/query

Zipf Model

- Terms $t_1, \ldots, t_m$
- To generate a document we take every t_i with probability $\frac{1}{i}$
- Database is n independently chosen documents
- Query document has exactly one term in every interval $[e^i, e^{i+1}]$
- Similarity between documents is defined as the number of common terms

Magic Level Theorem

Magic Level $q = \sqrt{2 \log_e n}$

Theorem (Hoffmann, Lifshits and Nowotka, CSR'07)

1. *With very high probability there exists a document in database having $q - \varepsilon$* **top** *terms of query document*
2. *With very small probability there exists a document in database having* **any** *$q + \varepsilon$ overlap with query document*

Part III
Proving Hardness of Nearest Neighbors

Inclusions with Preprocessing (1/2)

Input
Family $\mathcal{F}$ of subsets of U

Query task
Given a set $f_{new} \subseteq U$ to decide whether $\exists f \in \mathcal{F} : \quad f_{new} \subseteq f$

Constraints
Data storage after preprocessing $poly(|\mathcal{F}| + |U|)$
Time for query processing $poly(|U|)$

Open problem: is there an algorithm satisfying given constraints?

Inclusions with Preprocessing (2/2)

Reformulation in SAT style:

Input
DNF formula $\mathcal{F}$ on n variables, without negations

Query task
Given an assignment x to evaluate $\mathcal{F}(x)$

Constraints
Data storage after preprocessing $poly(|\mathcal{F}|)$
Time for query processing $poly(n)$

Open problem: is there an algorithm satisfying given constraints?

"NP Analogue" for Search Problems

Every problem in **SEARCH class** is characterized by poly-time computable Turing Machine M:

Input
Strings $x_1, \ldots, x_n, \qquad |x_i| = m$

Query task
Given string y of length m to answer whether $\exists i : M(x_i, y) = yes$

Tractable problems in SEARCH

Input
Strings $x_1, \ldots, x_n, \qquad |x_i| = m$

Query task
Given string y of length m to answer whether $\exists i : M(x_i, y) = yes$

Tractable solution
Preprocessing in $poly(m, n)$ space

Query processing in $poly(m, \log n)$ time with RAM access to preprocessed database

Inclusions is in SEARCH. Is it tractable?

Complete problems in SEARCH (1/2)

Program Search problem:

Input
Turing machines $P_1 \ldots, P_n$

Query task
Given string y of length m to answer whether $\exists i : P_i(y) = yes$ after at most m steps

Open problem: is Program Search tractable?

Complete problems in SEARCH (2/2)

Parallel Run problem:

Input
$x_1 \ldots, x_n$

Query task
Given poly-time computable P to answer whether $\exists i : P(x_i) = yes$

Open problem: is Parallel Run tractable?

Conclusions

Call for Feedback

- Any relevant work?
- How to improve this talk for the next time?
- **Give my open problems to your friends!**

Summary

- Nearest neighbors for texts can be modelled by Euclidean space with sparse vectors
- No exact algorithms for NN in texts are known so far
- New approach to lower bounds for NN: SEARCH class and its complete problems

Thanks for your attention! Questions?

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